Thank you, President Hill. It is an honor to be invited to address the Vassar community and its guests at today’s convocation. Convocation is the ceremonial beginning of the college experience for the Class of 2017, and the beginning of the final year of college for the Class of 2014. Usually it arrives most years with cooler nights and fresher air. In spite of today’s weather, convocation always energizes me, for a new academic year and all the promise such a year offers.

I hope that the picture title of my talk has drawn your attention. I have taken my cue from the performance artist, formerly known as Prince, now known once again as Prince. All will be explained shortly.

When you invite a mathematician to speak, he or she will try to follow certain rules. For example, the talk must include a proof of some sort. And it should include a problem or two for the listeners so they can make some new mathematics of their own. I find it disconcerting to talk about my experiences as a mathematician, without telling you all about the mathematics itself. Don’t worry, we will only do a little bit of math.

I was born seven years after the Second World War. My mother was born in Nagoya, Japan. She met my father, a U.S. soldier from Philadelphia, when he was stationed in Tokyo during the Korean War. They crossed the Pacific together in 1952 to settle in the Philadelphia area where I was born a couple of months later. When I went to elementary school, I found the students were quick with upsetting racial taunts directed at me. My mother told me to work at being a good student, and that was what was important at school. Perhaps I am here speaking to you today thanks to my mother’s wisdom.

I started college with three majors in mind—Theatre: I wanted to write plays, having acquired a fascination with the work of Samuel Beckett; Linguistics: I wanted to learn as many languages as I could, because, after exposure
to Latin and German, I hadn’t met a grammar I didn’t like, and there were so many new ways to say things; and I wanted to learn more Mathematics.

It was during sophomore year in a multivariable calculus class that I got an inkling that math might be the major for me. Professor O’Neill described a particularly complicated three-dimensional figure and I saw it plainly in my mind’s eye. I could rotate it in space, and I even believe I could tell you that it was brown! Further courses took me from computational mathematics to proofs and abstract theories, where I found a new landscape, rich in strange objects and described in a delightful new language: there were noncommutative groups, an infinity of infinities, uniform continuity, Moufang loops, and homeomorphisms. I no longer sought a right answer, but a right reason for believing in the truth of an assertion. And after finding the reason, I needed to craft a careful argument, in exacting prose, to establish the truth. I explained to a house-mate at that time how I had found in the pursuit of mathematical truths the kind of satisfaction I sought in writing a play or in discovering the right words in a foreign tongue.

I went directly on to graduate school, eager to deepen what I knew and to get to know new objects of study and the new language that described them. At the end of the first semester, in my favorite class, I had a real crisis of confidence. I was assigned a routine computation that I simply could not find a way to do. I was crestfallen, and admitted as much to my classmates. The Dutch returning student in my class suggested an idea that I realized I had learned as a freshman at college. She also suggested that I was taking myself too seriously. She was right on both accounts, and I carried on.

So how did I become a mathematician? The word ‘mathematics’ has its origin in the ancient Greek word µάθημα (máthêma), “that which is learnt,” or “what one gets to know.” Being in graduate school seemed to cover that part of it. There had to be more to it, a deeper experience I had yet to find.

What I was seeking has been described well by the Princeton mathematician, Andrew Wiles, in the PBS documentary, The Proof:

Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. One goes into the first room, and it’s dark, completely dark. One stumbles around bumping into the furniture, and gradually, you learn where each piece of furniture is, and finally, after six months or so, you find the light switch. You turn it on, and suddenly, it’s all illuminated. You can see exactly where you were.¹

So, you find a dark mansion to stumble around in, and hope that you can

¹From the transcript of the television program The Proof, produced by PBS, written by Simon Singh.
find the light switch! I was drawn to the vast mansion called topology, the house of rubber-sheet geometry, and particularly the wing called algebraic topology, where algebraic ideas—adding, subtracting, and multiplying objects—combine with geometry into a heady mix.

To get a PhD in mathematics you have to find an area of study, a dark mansion, and find a problem, preferably unsolved, and work away at solving it. If you don’t find the light switch, your thesis might be a description of some of the furniture in that dark room. It is usual for your PhD advisor to assign you a problem and to get you started. My advisor, Jim Stasheff, had the good sense to let me find my thesis problem for myself—a useful life skill. I found a phenomenon in one of his papers that I thought had an analogue in a different setting. After 3 or 4 tries I fashioned a tool, something called an obstruction theory, that worked nicely. What I didn’t have was a particular problem to which my tool might be applied. It was like having plans and a hammer, but no nails or wood. I read widely, and I remember a Saturday morning in Spring, watching cartoons, when it occurred to me that I could apply my tool to a class of well known examples by a simple trick. I had found my PhD thesis! My reaction at the time was an incredulous “Is it that simple?” I was certain that what took me so many months of work could have been done by anyone.

I was happy to have a PhD thesis, but surely making mathematics ought to have been more dramatic—Saturday morning cartoons after all. It was almost as if I had accidentally brushed the light switch that was plainly under my hand all along.

Other mathematicians have described their moments of clarity in dramatic stories. My subtitle is taken from one of the most well known accounts, described by Henri Poincaré:

Just at this time I left Caen, where I then lived, to take part in a geologic excursion organized by the École des Mines. The circumstances of the journey made me forget my mathematical work; arrived at Coutances we boarded an omnibus for I don’t know what journey. At the moment when I put my foot on the step the idea came to me, without anything in my previous thoughts having prepared me for it; that the transformations I had made use of to define the Fuchsian functions were identical with those of non-Euclidean geometry. I did not verify this, I did not have time for it, since scarcely had I sat down in the bus than I resumed the conversation already begun, but I was entirely certain at once. On returning to Caen I verified the result at leisure to salve my conscience.²

²From Science et Méthode by Henri Poincaré, first published in Paris in 1908 and translated
Let’s get a feel for what mathematicians experience by proving something together. The title of my talk in the printed program is a pair of squares of equal size. Each is subdivided into some grey squares and some right triangles. On the right, there is one grey square. Each of its sides is the hypotenuse of one of the four equal right triangles making up the rest of the square. On the left, there are two grey squares, the larger one has sides equal to the longer leg of the right triangle, and the smaller square has as its side the smaller leg of the right triangle. In the square on the left, there are also four copies of the same right triangle in the big square. Two by two they form the two rectangles next to the grey squares. Can everyone see all the bits of the figures?

Beginning with the two big squares, remove the four equal right triangles. We have removed equal parts from equal squares, and so the pieces that remain, the grey squares, are equal in area. Thus the square on the hypotenuse of the right triangle has the same area as the squares taken together on each of the legs of the triangle.

We have proved the Pythagorean theorem!

Notice the lack of $a^2 + b^2 = c^2$, in fact, the complete lack of symbols. This is a geometric argument of the best type: it is simple, it is irrefutable, and with it we have established a truth.

I hope that some of you experienced a little rush of pleasure from the proof. This argument is my bet for the proof known to the Pythagorean brotherhood. I get a thrill that we can connect directly with the culture of ancient Greece, 2500 years ago. We share not only the truth of the Pythagorean theorem but also the pleasure at establishing it. To quote the English mathematician G.H. Hardy, “there must surely be something to be said for a study which did not begin with Pythagoras, and will not end with Einstein, but is the oldest and the youngest of all.”

To give you the chance to make your own mathematics, here is a nifty problem. (You may need a writing instrument.) Choose a 2-digit number with digits that differ by at least two. For example, 17, or 14. Numbers like 12, 76, or 33 are excluded, so if you have picked a number like that, please pick another.
All set? Now reverse the digits to get a new 2-digit number. You have two 2-digit numbers now, so subtract the smaller from the greater. For example, if your number is 17, you get $71 - 17 = 54$. This will give you a new 2-digit number. Now add that number to its reverse. In my case, $54 + 45 = 99$. I get 99, and you get 99. In fact, this procedure always results in 99.

Why is that true? Of course, you could check every possibility (there are 63 cases) to prove that it is true in each case. You might want to try a little algebra to show that you always get 99. Perhaps you want to think about 3-digit numbers. Or even 4-digit numbers. These are the kinds of problems that will make you think like a mathematician.

Back to my story: Having found a thesis, and earned a doctorate, I landed a one-year teaching position in Maine. I wondered how to proceed as a mathematician. Although I had reached some plateau, what should I do next? I no longer had the supportive community of graduate school; my advisor, other topologists, and the stimulation of seminars, other students, and invited speakers. I also realized that, though my graduate studies were broad, and I knew a lot about what I had done in my thesis, I still hadn’t mastered some of the basic tools of my specialty. This fact was brought home to me when I gave a short talk in New York City on my thesis results. A tall, quiet man in the audience (Mark Mahowald) stated his opinion that obstruction theories were passé, and that I could do all this work better with a spectral sequence. I had learned a little about spectral sequences as a graduate student, producing some simple computations, and some failed computations. Spectral sequences are among the most feared and most complicated of mathematical objects. Here is a sample from the internet of feelings folks have about spectral sequences:

“The words ‘spectral sequence’ strike fear into the hearts of many hardened mathematicians.”

“They have a reputation for being abstruse and difficult. It has been suggested that the name ‘spectral’ was given because, like spectres, spectral sequences are terrifying, evil, and dangerous.”

They were also standard tools in algebraic topology, so I had to learn to use them. At the time, there wasn’t a good book focusing just on spectral sequences, so I naively decided to write one. After all, I could learn everything I needed in the writing, and if it were published, students of topology and algebra might get a leg up on these complicated objects. I had no idea how to write a book, and

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4Ravi Vakil in *Spectral sequences: Friend or foe?*
I had no idea that other mathematicians had given up on trying to write such a book. The grace of ignorance favored me, and after a few years, I finished it. My head was filled with algebraic topology, many wonderful arguments, giving me the key to the front door of one of the dark mansions Wiles described. I was finally ready to stumble around in the dark looking for a light switch.

The dark mansion I entered was built on a problem of interest in geometry. This geometric problem could be turned into a question about spectral sequences, making it an algebraic problem. According to Michael Atiyah, I had taken an offer made by the devil:

Algebra is the offer made by the devil to the mathematician. The devil says: ‘I will give you this powerful machine, it will answer any question you like. All you need to do is give me your soul: give up geometry and you will have this marvelous machine.’

Together with a colleague, I had made some successful computations bearing on the problem with certain spectral sequences. That led me to another problem: Did my glorious algebraic machines have sufficient input to have even a chance of settling the question we wanted to answer? I spent 4 months stumbling around in the mansion of Algebra searching for a new idea. The furniture in the mansion of Algebra is very ornate. I was spending a sabbatical year in Göttingen, Germany, at the university of Gauss, Riemann, Einstein, and Hilbert. The grad students there had improved my high school German considerably by the Spring, when a Canadian colleague, Richard Kane, visited for a week. We toured the restaurants and bars each evening for dinner and I played the role of tour guide and interpreter. We talked about mathematics, what we were working on, and what we had recently achieved. Before leaving for a weekend in Berlin, I attended Richard’s colloquium talk. He opened his lecture with a review of a little spectral sequence, the Bockstein, and I immediately realized that this tool was the key to the question I had worked on for the last few months. The argument that forced things to be small in Richard’s account could be reversed to produce lots and lots of algebra for my purposes.

I had found my light switch, I could see the room clearly, and it was exhilarating!

Every mathematician wants to have such a moment, and I feel privileged to have experienced it in my work life. Is it a frequent occurrence? One of my favorite living mathematicians, Jean-Pierre Serre, said that sudden discoveries had come to him only twice in sixty years.

Of course, such experiences are not restricted to mathematicians, but they can occur in any sort of creative process. Most of us, at one time or another,
are seeking that inspired moment, that flash of insight, that lifts us up to some new understanding of our humanity. Sometimes it arrives like a lightning bolt. Sometimes it accumulates like snow on a roof; when it is the right weight, it falls.

What can we learn from these experiences? I don’t recommend a crash course on spectral sequences as a means to extraordinary experiences. My experience suggests, first, get ready. Learn everything that you can, about your studies, about that open question, about that period of history, . . . . Discuss what you are learning with everyone—your professor, your classmates, your family and friends, the barista at the coffee shop. Learn the languages of discourse shared by the community of researchers.

Secondly, try to have lots of ideas while you are learning new things. Many of your ideas will fail, or fall short of your goal. But a good idea may be hiding in the failed attempts. So just have ideas and don’t worry about when they fail. I once got a phone call from Sweden. “Hallo, this is Jan-Erik Roos from Stockholm. Can you tell me if Lemma 2 in Chapter 3 of your paper is correct? I have a counterexample.” This is how I found out that I had published an error. After a mostly sleepless night, I could account for the error and explain Roos’s example, and later I wrote a correction for the journal. I am still playing with the new things I learned that night. Failures are sometimes essential to learning.

A third thing I learned from lightning experiences is that you have to be open to them. Flashes of inspiration can be unexpected. I have met many fellow travelers who always want to learn the next thing, who claim that they cannot be prepared without reading another book or article. When you are trying to explore a problem with this approach or that approach or the other theory, all that competition for your attention can drown out the quiet voice of inspiration. I found an apt description of this disposition in Frank Delaney’s novel, Ireland. In this excerpt, a poet describes his art:

As you probably know, nobody can actually write a poem. There’s no such act as writing a poem. That’s not how poems are made. Oh, yes, there’s the physical business of pen, ink, and paper—but that isn’t whence the poem comes. Nor may you send out and fetch a poem from where it’s been living. No, like it or like it not, you have to wait for a poem to arrive.

The people we call “poets,” by which I mean true, real poets—they’re merely very keen listeners who’ve learned to recognize when a poem’s dropping by. Then they copy down what the poem’s telling them in their
heads. ...\textsuperscript{5}

Find your passion, or in some cases, passions, learn everything you can about it, learn to talk about it, and have ideas, then listen to what your passion whispers to you. I wish you all success in this and I hope that you too will have the exhilarating experience of inspiration. It can happen any time if you are ready, in small ways and in big ways. Prepare to step onto the bus.